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Influence of the thickness and polarization charges on the electro-optical behaviour of twisted smectic C* cells

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This paper reports on the influence of both polarization charges and thickness on the light transmitted by twisted smectic C* (TSC) liquid crystal (LC) cells. A uniaxial model, based on the de Vries eigenmodes usually used for twisted structures like cholesteric LCs, is used to calculate the transmission variation of the TSC cells as a function of applied voltage or thickness for various values of spontaneous polarization. The results obtained are in quantitative agreement with experimental results found in the literature and emphasize the importance of polarization charges on the behaviour of TSC cells. The TSC light transmission exhibits a profile resembling the so-called V-shaped switching.

1. Introduction

Since 1980, surface-stabilized ferroelectric liquid crystals (SSFLC) have been widely investigated. They have several advantages over conventional nematics such as high speed of operation and bistable device configuration [1–5], but they lack grey scale capability. In 1992, Patel proposed an electro-optical device (the twisted smectic C* cell) with a high capability of producing grey scale and a relatively high switching speed [6]. This liquid crystal light modulator uses a ferroelectric smectic C* liquid crystal with large tilt angle ($\theta \simeq 45°$). The cell is made up by two conductive surfaces treated to achieve a planar orientation of the molecules. The two electrodes are rubbed at an angle of 90° with respect to each other. The smectic layers are perpendicular to the cell surfaces and the angle between the layers and the rubbed axis is 45°. If strong anchoring is assumed, the molecules are coplanar on the cell electrodes and deviate from the cell plane in the bulk. The projection of the molecules, on a plane parallel to the cell surfaces, rotates by an angle of $2\theta = 90°$ from one electrode to the other and the ferroelectric polarization by an angle of 180°. The cell is placed between crossed or parallel polarizers. Optical transmission through the cell is driven by the applied voltage, since the ferroelectric dipoles tend to align with the electric field inside the cell.

In the first studies devoted to the TSC, the electric field caused by the polarization charges were not taken into account [7, 8]. However the spatial divergence of the spontaneous polarization vector locally induces polarization charges in space [9–11]. The resulting polarization field affects the spatial structure of the smectic C* LC. To date, only a few authors have taken into account the effect of these charges on the optical behaviour of the TSC. Using the well known Jones matrix method, Guena [12] calculated the transmitted light intensity by the TSC, and Rudquist *et al.* [13] calculated the optical transmission of a V-shaped switching cell by using the Berreman 4×4 matrix method.

In this work, we study the influence of the polarization charges and thickness of the cell on the transmission of light by TSC cells. To calculate the transmission variation versus applied voltage, we use a uniaxial model based on the well known de Vries eigenmodes [14]. These calculations require a knowledge of the azimuthal angle variations from one side of the cell to the other for different applied voltages.

2. Azimuthal distribution of molecules inside the cell

We consider that the molecules and the polarization vector are strongly fixed to the two conductive surfaces of the cell. Between the lower and upper surfaces of the cell, the molecules rotate continuously on the smectic C tilt cone (figure 1). As a consequence the spontaneous polarization has a variable orientation through the cell. Therefore the projection of the molecules on the plane of the cell exhibits a 90° rotation angle and the polarization undergoes a rotation of 180° along the z direction of a cartesian coordinate system. The reversal of $\mathbf{P}_s$

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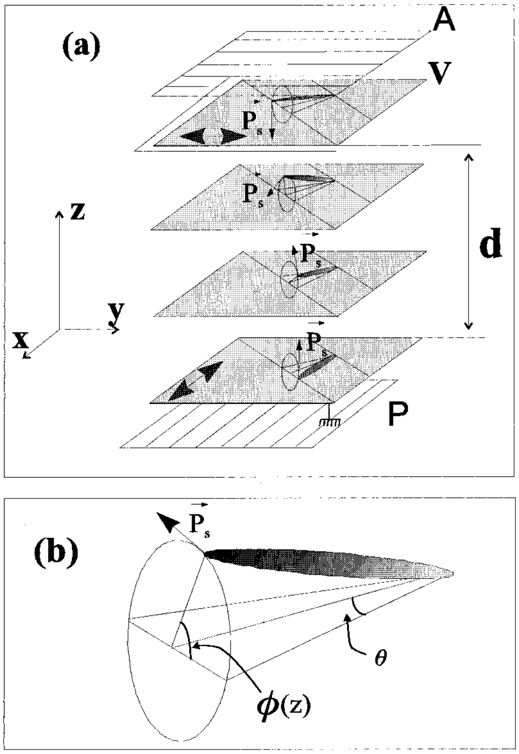

Figure 1. (*a*) Rotation of the molecules between the cell faces and related modifications of the spontaneous polarization ($\mathbf{P}_s$). (*b*) Tilt angle θ and azimuthal orientation ϕ. P = polarizer, A = analyser, d = cell thickness, V = Applied voltage, $\mathbf{P}_s$ = spontaneous polarization, θ = tilt angle, ϕ = azimuthal angle.

leads to the appearance of surface charges on the electrodes with the density $\sigma_p = -\mathbf{P}_s$ (figure 2). The deformation of $\mathbf{P}_s$ leads to charges with volumic density $\rho_p = \mathbf{P}_s(\mathrm{d}\phi/\mathrm{d}z)\sin\phi$, where ϕ represents the azimuthal angle which is only z dependent. The electrical densities σ_p and ρ_p have opposite signs and the induced charges create internal electric fields that combine with the external field resulting from the applied voltage. The symmetry of the problem suggests that the director and the electric field depend only upon the z coordinate. Therefore the internal electric potential can be determined by using the Poisson equation.

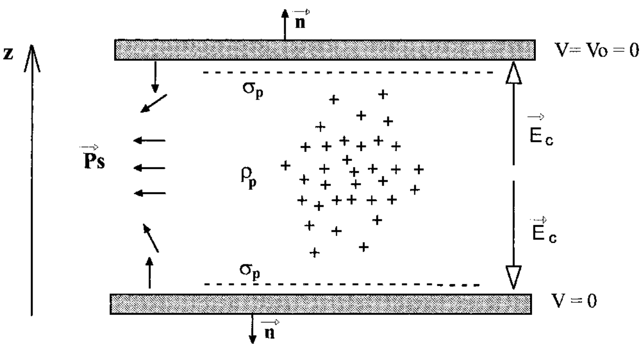

Figure 2. Influence of the polarization charges on the molecular azimuthal distribution $\phi(z)$ in the absence of applied voltage, $V_0 = 0$.

In studies devoted to the TSC [6–8], the dielectric contribution to the free energy is neglected when comparing its magnitude order to the elastic energy of the cell and to the interaction energy between the electric field and the spontaneous polarization. However, in thin samples ρ_p does not necessarily vanish and the polarization field should affect the spatial structure and therefore the TSC LC cell properties. According to Nakagawa and Akahane [9], taking into account the elastic properties, the dielectric properties and the interaction between electric field and polarization energies, the free energy minimization leads to a system of two equations:

$$\varepsilon_{zz}\left(\frac{\mathrm{d}^2 V}{\mathrm{d}z^2}\right) + \mathbf{P}_s \sin\phi\left(\frac{\mathrm{d}\phi}{\mathrm{d}z}\right) = 0 \qquad (1)$$

$$A\left(\frac{\mathrm{d}^2 \phi}{\mathrm{d}z^2}\right) + \mathbf{P}_s \sin\phi\left(\frac{\mathrm{d}V}{\mathrm{d}z}\right) = 0 \qquad (2)$$

where $\varepsilon_{zz} = \varepsilon_0 \varepsilon_r$ is the dielectric constant (the dielectric anisotropy is neglected [9]), V is the electric potential and the constant $A = K\sin^2\theta$ is obtained when considering the same elastic constant (K) in all directions.

The equations (1) and (2) can be numerically solved in the case of strong anchoring with the following surfaces conditions:

$$\phi(0) = 0, \quad \phi(d) = \pi$$
$$V(0) = 0, \quad V(d) = V_0$$

where V_0 is the applied voltage and d the thickness of the cell.

For ease of comparison, our computations are based on the same values used in the pioneering work [8, 12]. Parameters used for the calculations are:

Tilt angle	$\theta = 43°$
Dielectric constant	$\varepsilon_r = 13$
Wavelength	$\lambda = 0.633\,\mu\mathrm{m}$
Ordinary index	$n_o = 1.6$
Extraordinary index	$n_e = 1.72$
Constant	$A = K\sin^2\theta$ $= 8.1 \times 10^{-11}$ N
Spontaneous polarization	$\mathbf{P}_s = 72\,\mathrm{nC\,cm^{-2}}$

We choose the lower face voltage as the reference $V = 0$. At the upper face, the applied voltage $V = V_0$ can be adjusted between -3 and $+3$ V. When computed with

Matlab software, the system of equations (1) and (2) furnishes two sets of curves where the voltage appears as a parameter:

(1) Figure 3 gives the electric potential distribution through a cell of thickness 3.1 μm chosen for calculation as in previous work [8, 12]. For each external voltage that appears as a parameter, the corresponding curve leads to the effective local electric field inside the cell.

(2) Figure 4 describes the azimuthal distribution of molecules through the cell. When no voltage is applied to the cell ($V_0 = 0$), the electric potential and the azimuthal angle distributions are symmetrical in comparison with the middle of the cell. When $|V_0|$ increases, the twist is expelled towards one electrode and becomes nearly uniform in the rest of the cell. The cell behaviour cannot be described only by simple models such as a wave guide Mauguin regime [15] that requires a weak twist of the LC sample or a regime of linear birefringence because of the highly twisted cell regions. For this reason, we used the Mauguin–de Vries eigenmodes which can take into account any value of the local twist.

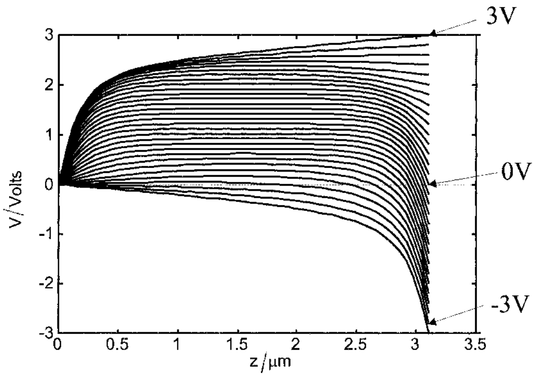

Figure 3. Potential distribution through the cell for various values of applied voltage V_0: (from − 3 V to 3 V in steps of 0.2 V). The cell thickness is $d = 3.1$ μm.

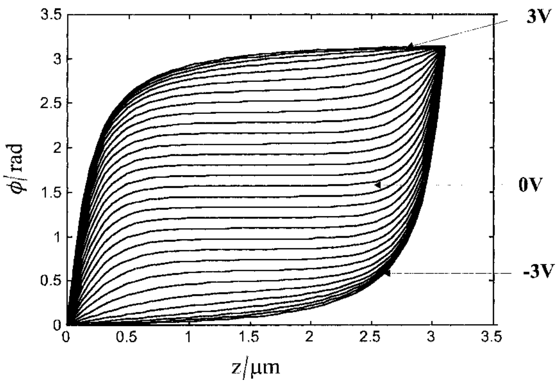

Figure 4. Azimuthal angle distribution inside the cell ($d = 3.1$ μm) for different applied voltages.

3. Light transmission through the cell

In planes parallel to the cell faces, the molecular distribution is assumed to be uniform. Owing to the azimuthal distribution against thickness, the refractive indices can be calculated in each plane $z =$ constant of the cell. The apparent rotation of the molecules between the z and $z + \mathrm{d}z$ planes enables estimation of the local pitch of the slice. Then the propagation of light through the cell can be performed using finite element technique: slices in which the pitch is assumed to be constant are defined. In the plane of the slice, the refractive index is given by the formula:

$$n(\theta, \phi(z)) = \sqrt{\frac{n_e n_o}{[n_o^2 + (n_e^2 - n_o^2)\sin^2\theta\sin^2\phi(z)]^{1/2}}}$$

where n_o and n_e are, respectively, the ordinary and extraordinary refractive indices.

The rotation angle dα (figure 5) related to the thickness variation dz defines the local pitch $p(z)$:

$$\mathrm{d}\alpha = \arctan[\tan\theta\cos\phi(z)] - \arctan[tan\,\theta\cos\phi(z + \mathrm{d}z)]$$

and

$$p(z) = 2\pi\frac{\mathrm{d}z}{\mathrm{d}\alpha}.$$

On entry to a slice, the incident light is decomposed on the two de Vries eigenmodes [14] relative to the

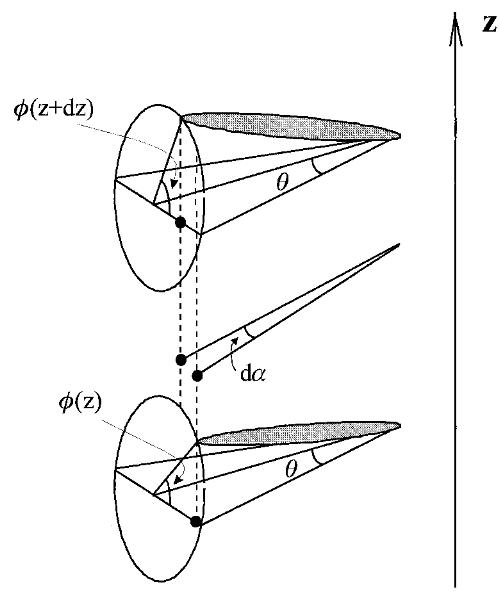

Figure 5. The apparent rotation dα of the director for a slice of thickness dz.

local pitch and the local indices. As extensively described in the literature, the eigenmodes have different elliptical polarizations and are associated with propagation indices [15, 16] that also depend upon the local pitch and local refractive indices of the slice. Taking into account the phase dependence of each eigenmode during crossing of the slice, the exit vibration is calculated. This then becomes the incident vibration for the following slice. So the azimuthal rotation through the cell is taken into account. Finally the transmission of the whole cell between the polarizers is calculated.

Figure 6 is drawn for a cell of 3.1 μm thickness. The polarizer is set parallel to the rubbed direction of the entry face of the cell. The extreme transmission—a maximum between crossed polarizers and a minimum between parallel polarizers—is obtained when there is no applied voltage ($V_0 = 0$). This can be easily understood: the directions of the slow or fast vibrations, i.e. the neutral lines of the cell, are then quasi parallel or perpendicular to the normal of the smectic layers and the polarizer is oriented at $\theta = 45°$ to these lines (the situation would be different with other values of θ and/or other orientations of the polarizer [13]). When a voltage is applied, this curve shows that (for crossed polars for instance) the transmission decreases quickly from its maximum to a value close to 0 for $|V_0| \simeq 3$ V. The same behaviour (extreme value for $V_0 = 0$) is obtained for other thicknesses. The extreme value depends on the thickness. Presently, our method furnishes a better fit with the experimental study of the cell of thickness 3.1 μm than the Pertuis method [7]. The remaining difference between the experimental results and our calculated results has to be sought in the existence of chevrons that often appear in such thin cells.

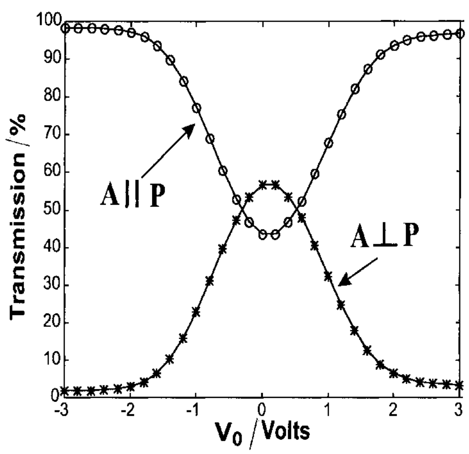

Figure 6. Transmission against voltage for parallel and crossed polarizers obtained for a cell of thickness $d = 3.1$ μm and $\mathbf{P}_s = 72$ nC cm^{-2}. The polarizer is set parallel to the rubbed direction of the entry face.

4. Influence of the cell thickness on the optical transmission of the TSC

In order to find the cell thickness which gives the maximum in contrast (optimal thickness) with given physical parameters of the LC compound, we now study the transmitted light intensity variations against thickness in the absence of an applied voltage. For each cell thickness, we calculate the azimuthal angle distribution, then the local pitch and the local indices for each slice according to the chosen step for numerical computations. The propagation through each slice is performed with appropriate de Vries eigenmodes as described above. Figure 7 indicates the transmission versus thickness that can be reached when the polarizers are crossed or parallel. With the above defined properties values for the ferroelectric compound (see § 2), optimal thicknesses are found at 5.4 μm and $\simeq$ 16 μm. Notice that for thick samples, ρ_p must vanish, because the natural twisting of the smectic C* forbids the bookshelf geometry situation. Figure 8 depicts the transmission variations against applied voltage for a TSC cell of optimal thickness

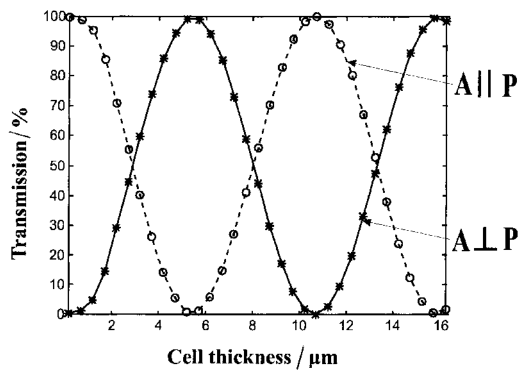

Figure 7. Transmission against thickness for $V_0 = 0$ V and $\mathbf{P}_s = 72$ nC cm^{-2}.

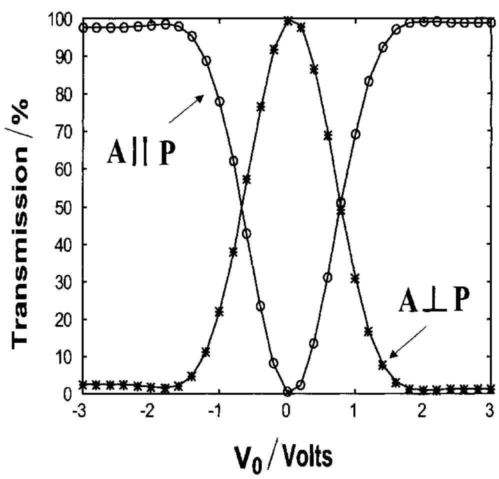

Figure 8. Transmission versus applied voltage for a twisted ferroelectric cell of thickness $d = 5.4$ μm and $\mathbf{P}_s = 72$ nC cm^{-2}.

$d = 5.4\,\mu m$. This curve shows that the contrast variations can occur in a quasi-linear way. Notice that the profile obtained resembles the so-called V-shaped switching profile [17]. Numerical computations enable us to observe the influence of various parameters of the compound used. For instance, the variations of transmission against thickness for different values of the spontaneous polarization can be studied (see figure 9 which is obtained in the case of crossed polarizers). We can note that for low spontaneous polarization, the optical transmission versus thickness oscillates as for the well known 90° twisted nematic case [18]. When the thickness is large, the amplitudes of the oscillations decrease and the transmission reaches 100%; the contrast is then optimized for any thickness. When the thickness is smaller, the transmission oscillates between 100% maxima and minima; the optimal thicknesses have discrete values, the shortest being close to $10\,\mu m$. When the polarization increases, the amplitudes of oscillations become larger and the optimal thickness decreases to reach $d = 5.4\,\mu m$ for $\mathbf{P}_s = 72\,nC\,cm^{-2}$. Similar behaviour of the transmission is obtained for parallel polarizers (figures 7 and 10). The calculated transmission versus applied voltage for $\mathbf{P}_s = 5\,nC\,cm^{-2}$ and a cell thickness of $10.4\,\mu m$ is given in figure 11. Compared with figure 8, figure 11 shows, on the one hand, that higher voltage values are needed to achieve a V-shaped-like response and, on the other hand, that the transmission has lost its linearity. Finally, taking into account the real values of the physical parameters of the LC compound used, such as elastic constants and dielectric anisotropy, the required thickness for optimal transmission of light can be determined for any LC cell.

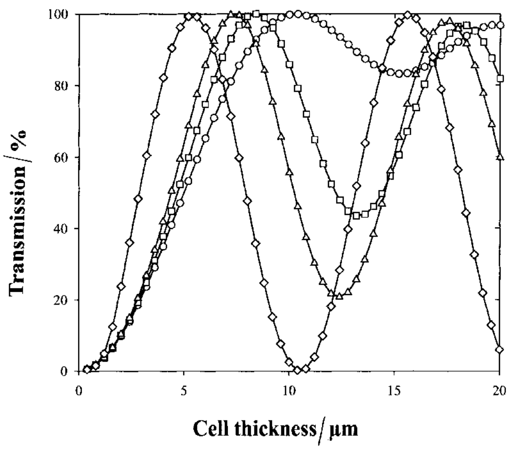

Figure 9. Transmission against thickness at $V_0 = 0$ V for various values of the spontaneous polarization when the polarizers are crossed. (○) $\mathbf{P}_s = 5\,nC\,cm^{-2}$, (□) $\mathbf{P}_s = 7.5\,nC\,cm^{-2}$, (△) $\mathbf{P}_s = 10\,nC\,cm^{-2}$, (◇) $\mathbf{P}_s = 72\,nC\,cm^{-2}$. The optimum thickness evolves between a half-wave plate for strong polarizations and a wave plate for weak polarizations.

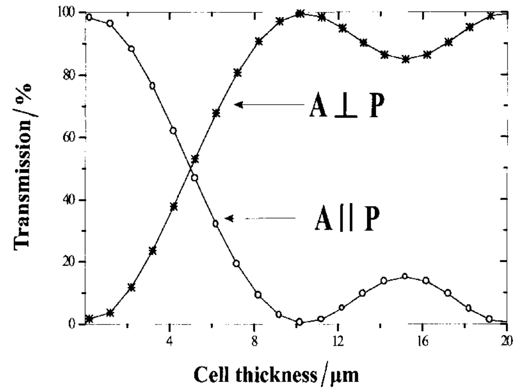

Figure 10. Transmission against thickness at $V_0 = 0$ V obtained for a compound of low polarization ($\mathbf{P}_s = 5\,nC\,cm^{-2}$).

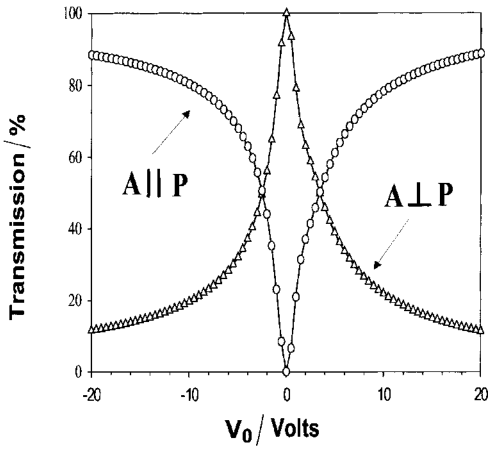

Figure 11. Transmission versus applied voltage for a twisted ferroelectric cell of thickness $d = 10.4\,\mu m$ and $\mathbf{P}_s = 5\,nC\,cm^{-2}$.

5. Discussion and conclusion

From the above studies, a schematic analysis of the TSC LC cell behaviour can be drawn. As the $\mathbf{P}_s$ becomes large, so the influence of polarization charges becomes important. Except near the cell faces, the director orientation is nearly homogeneous inside the cell. The smectic C* liquid crystal then acts as a birefringent plate that rotates according to the applied voltage. The equivalent birefringent plate low axis is along the polarizer ($\alpha = 0$) when $V_0 \simeq -3$ V, is oriented at 45° from the polarizer when $V_0 = 0$ and is perpendicular ($\alpha = 90°$) to the polarizer when $V_0 \simeq 3$ V. With linearly polarized incident waves, the best contrast is obviously obtained when the cell acts as a half-wave plate (this is also true whatever the polarizer orientation, because the transmitted light intensity varies as $\sin^2 2\alpha \sin^2(\pi\delta n/\lambda)d$). A π phase retardation is then obtained with a cell of thickness $d = \lambda/2\delta n$ where $\delta n \simeq \Delta n \cos^2\theta = \Delta n/2$ if $\theta = 45°$; $\Delta n = n_e - n_o$. If no voltage is applied, the required thickness is then approximately equal to $\lambda/\Delta n$ ($\approx 5\,\mu m$ with our numerical values).

In the opposite case, when $\mathbf{P}_s$ is small, the influence of polarization charges is weak. With an applied voltage, the cell also acts nearly as a homogeneous birefringent plate with neutral lines parallel or perpendicular to the polarizer. But when $V_0 = 0$, the director rotates uniformly through the cell. The cell is then inhomogeneous and the slow neutral line (projection of the optical axis) orientation inside the cell rotates from 0 to 2θ ($\simeq 90°$) according to the approximative law $\alpha = \theta(1 - \cos\phi)$. The best contrast is now obtained for any thickness values when the propagation through the cell in the '$V_0 = 0$ state' obeys the wave guide regime, i.e. when the thickness is much larger than the Mauguin limit ($d \gg \lambda/2\delta n$). But when the thickness approaches the Mauguin limit, the transmitted light is strongly elliptically polarized if the cell phase retardation is equal to an odd multiple of π and is linearly polarized if the phase retardation is a multiple of 2π. The contrast is then again optimized for discrete values of the thickness for which the phase retardation is a multiple of 2π. As $\delta n(z)$ is approximately equal to $\Delta n/2$, the first thickness leading to an extremum of the transmitted light is $d = 2\lambda/\Delta n$ ($\approx 10\,\mu m$ with our numerical values).

Between these two extreme situations ($\mathbf{P}_s$ large and $\mathbf{P}_s$ weak), the numerical computations presented here are needed and are able to take into account the optical activity in the strongly twisted regions near the cell faces (see curves obtained for $\mathbf{P}_s = 7.5$ and $10\,nC\,cm^{-2}$ in figure 9).

In conclusion, our method of calculation points out the role played by polarization charges in ferroelectric twisted cells [9–13]. In such a way, the behaviour as a function of thickness of cells filled with a compound with given properties can be studied in detail. The profile obtained sets out the possibility of getting a linear grey scale with SmC* twisted cells.

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